Algebra II

– Edition B –

(TwoPLUS for Maths Teaching)

Sample pages!

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Contents

Introduction

Algebra II is a maths project about Algebra, for intermediary level, 12-14 year old students. It deals with a range of new topics, while still giving you a chance to revise, consolidate and improve on your basics. You will train and extend your mathematical abilities, while special emphasis will be put on your personal preferences and gifts.

Algebra II is split into four main sections.

The **'Exercises'**:
You've really got a lot to calculate here. Each side is devoted to one single aspect of the topic. And the exercises will get gradually more diverse - and harder - as you go on. Roughly the first half of each side is given over to **basic exercises**. The second half contains lots of **extension exercises**. These are a bit more demanding mathematically. The '***star exercises**' belong to this second group. Sometimes they need a longer calculation, sometimes a special idea, sometimes .... ?! You'll use your lesson time to work through the exercises.

The **'PLUS-Activities'**:
That means fun and work! You can choose activities that seem particularly interesting to you or you can do short projects, that are built on your personal abilities and gifts. **Keywords** at the start of an activity help make your choice easier, by telling you what kind of activity it is. There are also 'hard nuts to crack' for the Maths geniuses amongst you.

The **'Further exercises'**:
These are intended for homework or to assess how you are achieving.

The **'Supplementary exercises'**:
These include additional consolidation of the key points. Your teacher might go through these sections with you or leave them out altogether.

Algebra II is available in two editions:

B – Student edition (workbook with answer key)
C – Teacher edition (includes the workbook (=Vol.1) plus a book of educational guidelines for lesson preparation (=Vol.2))

If you find any mistakes, or if you have any comments or questions, please contact me:

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So without further ado, allow me to wish you a lot of fun and successful learning with *Algebra II*,

Frank Rothe

Binomial formulae: Removing the brackets and factorizing
Exercise 7

1. Calculate the square numbers with help from the binomial formulae!

a) $13^2 = (10+3)^2 =$
b) $14^2 =$
c) $15^2 =$
d) $16^2 =$
e) $17^2 =$
f) $18^2 =$
g) $19^2 =$

Basic exercises

2. Simple examples using the three binomial formulae!

a) $(a+6)^2 =$
b) $(a+4)^2 =$
c) $(a+1)^2 =$

d) $(a-3)^2 =$
e) $(a-11)^2 =$
f) $(a-7)^2 =$

g) $(a+12)(a-12) =$
h) $(a-13)(a+13) =$
i) $(a-19)(a+19) =$

j) $a^2+14a+49 =$
k) $a^2+16a+64 =$
l) $a^2+20a+100 =$

m) $a^2-10a+25 =$
n) $a^2-30a+225 =$
o) $a^2-60a+900 =$

p) $a^2-400 =$
q) $a^2-81 =$
r) $a^2-196 =$

3. Fill in the binomial formulae! Watch out for the signs!

a) $(\ldots\ \ \ldots)^2 = a^2 + \ldots + 81$
b) $(\ldots + \ldots)^2 = \ldots\ \ 20a + 100$
c) $(\ldots\ \ \ldots)^2 = a^2 - 24a\ \ \ldots$
d) $(\ldots - \ldots)^2 = a^2\ \ \ldots + 16$
e) $(a + \ldots)(\ldots\ \ \ldots) = \ldots - 121$
f) $(\ldots\ \ \ldots)(\ldots\ \ 5) = a^2\ \ \ldots$
*g) $(\ \ \ 3)^2 = \ldots - 6a\ \ \ldots$
**h) $(6a + \ldots)^2 = \ldots\ \ 12a\ \ \ldots$

***4. Further examples using the three binomial formulae!**

a) $(4a+3)^2 =$
b) $(a+7b)^2 =$
c) $(7a-10)^2 =$
d) $(a-17b)^2 =$
e) $(9a+12)(9a-12) =$
f) $(a-13b)(a+13b) =$
g) $(4u+8v)^2 =$
h) $(18x-5y)^2 =$
i) $(11a+3+4a)^2 =$
j) $(a^2-2)^2 =$
k) $(3a^3-3b)(3a^3+3b) =$
l) $100a^2+84ab+96a^2+9b^2 =$

m) $9a^2+48a+64 =$
n) $a^2+26ab+169b^2 =$
o) $25a^2-180a+324 =$
p) $a^2-32ab+256b^2 =$
q) $144a^2-9 =$
r) $a^2-289b^2 =$
s) $81u^2+126uv+49v^2 =$
t) $225x^2-90xy+9y^2 =$

u) $a^8-10a^4+25 =$
v) $16a^4+16a^2+4 =$

Extension exercises

Exercise 12

1. **a)** **First simplify the** (original) **algebraic fraction!**

b) **Then subsitute the values for a into the original and into the simplified fraction!**

	original fraction	simplified fraction
	$\frac{a^2+8a+16}{a^2+4a} =$	_____
$a = 3$	_____ = _____	_____
$a = 4$	_____ = _____	_____
$a = 5$	_____ = _____	_____

Basic exercises

2. **Simplify the algebraic fraction!** Find a common factor or use a binomial!

a) $\frac{a^2+11a}{7a+77} =$

b) $\frac{ab+2b}{b^2+7b} =$

c) $\frac{2a^4+10a}{a^2b+4a^2} =$

d) $\frac{5a^2+45a}{10a+90} =$

e) $\frac{4a^2+4a+12}{a^2b+ab+3b} =$

f) $\frac{a^2+ab+3a}{2a^2+2ab} =$

g) $\frac{a^2+12a+36}{ab+6b} =$

h) $\frac{3ab+30b}{a^2+20a+100} =$

i) $\frac{5a^2-25a}{a^2-10a+25} =$

j) $\frac{a^2-2a+1}{8a^2b-8ab} =$

k) $\frac{a^2-49}{a^3-7a^2} =$

l) $\frac{a^2-16}{a^2b+4ab} =$

***3.** **Simplify the algebraic fraction!** Find a common factor or use a binomial or a trinomial!

a) $\frac{x^2+14x+49}{x^2+12x+35} =$

b) $\frac{11x^2-121x}{x^2-121} =$

c) $\frac{x^2+13x+40}{x^2+16x+64} =$

d) $\frac{9x^2-25}{9x^2+30x+25} =$

*e) $\frac{x-13y}{x^2-26xy+169y^2} =$

*f) $\frac{x^2-14x}{196-28x+x^2} =$

*g) $\frac{2x^2+12x-54}{324+72x+4x^2} =$

*h) $\frac{3x\cdot(x-15)^3}{x^3-30x^2+225x} =$

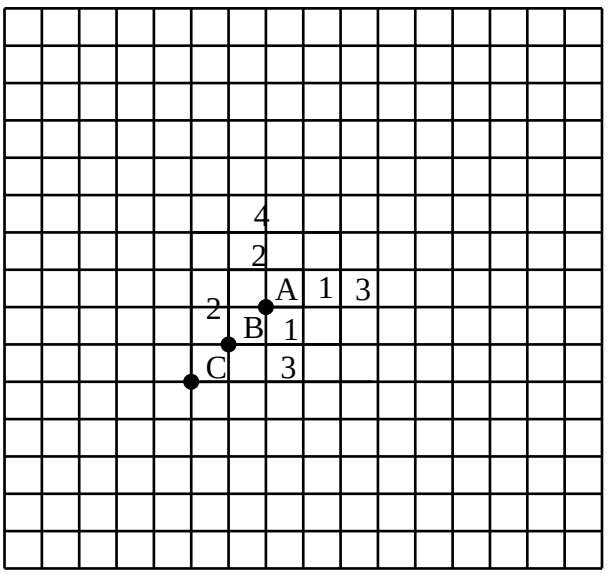
**i) $\frac{4x^2+32x+xy+8y}{x^2-x+8+10x} =$

**j) $\frac{x^4+12x+2x^3y+24y}{x^6+24x^3+144} =$

Extension exercises

Activity: Spirals

1. Spirals are always made up out of a number of whorls and channels. The first whorl begins exactly in the middle at point A and goes to point B. The second whorl begins at B and goes to C, and so on. The numbers tell us the length of each side. If you wish to, you may draw the continuation of the spiral.



Look at it exactly! How many sides does each whorl have?

a) How long is the spiral? Always start counting from the first (= the innermost) whorl. Fill in the table (completely)!

number of whorls	length of whorls	
1 (that means to B)	1, 1, 2, 2	= 6 (box lengths, 1BL=0.5 cm)
2 (that means to C)	1, 1, 2, 2, 3, 3, 4, 4	=
3		
4		
10		

b) **The formula!** Look for a formula to calculate the length of the spiral! If you know how many whorls the spiral has, you can calculate its length with the help of the formula. Explain your formula

c) Check your formula for 3 and 4 whorls.

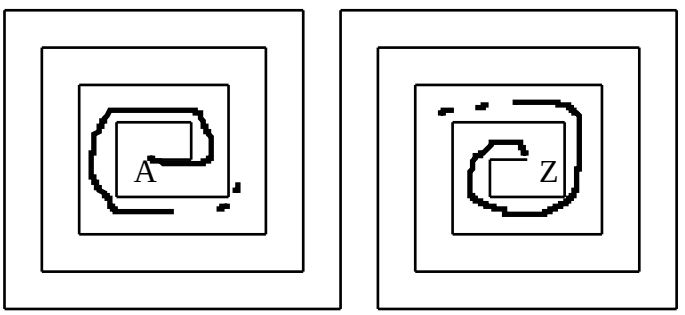
d) How long is a spiral with 100 whorls?

e) How long is the 100th whorl?

f) **Puzzle it out!** A spiral is 820cm long. How many whorls is it made up of?

Multiple intelligences

2. **Visualize!** In the double spiral, (see diagram, right), a rope is attached to startpoint A, pulled through the whole spiral, pulled tight and bound onto endpoint Z. The start and end points of the not yet tightened rope are already shown. How long is the rope exactly, when it is properly tightened? Imagine the boxes again (1 BL = 0.5cm). Describe how you calculated!



3. **Express yourself through ... !** At the edge of a town a park should be laid out in the form of a spiral. How would you design the park? Because of the location on the edge of the town you've got enough space available. You don't have to think about the cost at present. Think first of all about which groups of people could come to the park ... children ... older people ...
Which plants and flowers would you lay out?
Where do you plan to have areas of grass?
Would you have benches?
What could the visitors enjoy in the park?
What other elements would you like to have?
How are you going to put all this into practice?

Learning by using various expression style preferences

a) Set down a sketch of your park! Draw your ideas in!

b) Next, draw a plan of the park, to scale. Think first about how big the plan needs to be, so that there's enough room for your park, i.e. for your ideas. Now think about a suitable scale, so that the plan, (drawn to scale), fits on the paper. Do what needs to be done to finish your complete plan for the park.

4. **Express yourself using ... words!** A welcoming information board will be put up at the entrance to 'Spiral park' (see Nr. 3.). Develop a small poem or a short story that fits especially with the park to go on the board.

5. **Express yourself with ... your hands!** Produce a small piece of jewellery, that takes its inspiration from spirals. First draw a draft of the design. Which materials would you like to use? Are the materials suitable? Where can you get them from? And what do they cost? Which tools do you need? Where can you carry out the work? Do you need any further help? Who could help you further?

6. **Explore the subject!** Spirals occur in nature and technology. They often appear there under the name of 'spatial spirals'.
   a) In which concrete instances do spirals occur? Give examples and inform yourself more deeply about these.
   b) Take a closer look at the examples you've found. What differentiates the spirals from one another? Would you say that there are different kinds of spirals? How would you put these differences into words ... ?
   c) Make a poster giving a summary of the different types of spiral you've discovered, sorted systematically. Which information *has to be* on the poster?

7. **Express yourself through ... movement!** Develop a small dance sequence titled 'spirals'. What is important for you here? How can you translate the theme 'spirals' into an artistic performance? How do you want to build up the movement sequence? What about body position and facial expressions? Costumes? Musical accompaniment? Aim: give a short performance of your dance sequence.

8. **Express yourself through ... music!** Compose a short piece with the theme of 'spirals'. What are the characteristic elements of spirals? How could you translate that into music? Aim: a performance of your composition!

Integrating multiple intelligences

Supplementary exercise: **Algebraic division**

Supplementary exercise 15

1. **Divide!**

a) $30a^3 \div 6a =$ b) $28a^2 \div 7a =$ c) $18a^3 \div 3a^2 =$

d) $24a^3 \div 2a^3 =$ e) $15a^2 \div 5 =$ f) $36a^2 \div 9a =$

g) $(16a^3) \div (-8a) =$ h) $(-40a^2) \div (2a) =$ i) $(-30a^3) \div (-15a^2) =$

j) $(-9a^4) \div (3a^2) =$ k) $(-22a^2) \div (-2a^2) =$ l) $(36a^4) \div (-9a) =$

m) $\dfrac{30a^4}{15a} =$ n) $\dfrac{32a^3}{-8a^2} =$ o) $\dfrac{-27a^3}{9a} =$

p) $\dfrac{-48a^2}{-24a^2} =$ q) $\dfrac{20a^3}{-4a} =$ r) $\dfrac{-10a^4}{-20a^4} =$

s) $(15a+6) \div 3 =$ t) $(12a^2+8a) \div 4 =$

u) $(6a^2+14a) \div 2a =$ v) $(35a^3+10a^2) \div 5a =$

w) $(14a^3+21a^2+49a) \div 7a =$ Check: $7a \times (\quad) =$

x) $(18a^3+12a^2+6a) \div 6a =$ Check: $6a \times (\quad) =$

***2.** **Divide!** Now [illegible]

a) $(18ab^3) \div (6ab) =$ b) $(-44a^4b^2) \div (-11a^2b) =$ c) $(-20a^2b^3) \div (-4a^2b) =$

d) $(24a^7) \div (8a^3) =$ e) $(-32a^{10}) \div (8a^6) =$ f) $(-35a^8) \div (-7a^2) =$

g) $(-48a^3b^9) \div (-24a^2b) =$ h) $(12a^5b^2) \div (3a^2b) =$ i) $(-40a^7b) : (-20a^3b) =$

j) $\dfrac{-3a^3b}{ab} =$ k) $\dfrac{15a^4b^2}{-3a^4b} =$ l) $\dfrac{-21a^3b^3}{-7a^2b} =$

m) $\dfrac{2a^2b}{10a^3b} =$ n) $\dfrac{-6a^4b^2}{7a^4b^8} =$ o) $\dfrac{-a^2ba}{-a^3b} =$

p) $(21a^5+6a^2) \div 3a =$ q) $(12a^2b+44ab^3) \div 4ab =$

r) $(16a^5b^3+8a^3b) \div 8a^2b =$

s) $(6a^2b^4+10a^3b^5) \div 2ab^4 =$ Check: $2ab^4 \times (\quad) =$

t) $\dfrac{(21a^9b^7)}{(7a^6b) \times (-3a^3b^6)} =$ u) $\dfrac{(-4ab^7c) \times (9a^8b^2c^3)}{(-a^3b^9c^2) \times (-12a^4c)} =$

**v) $\dfrac{6a^3+24ab}{3a} =$

**w) $\dfrac{12a^4b^3-28a^5b^5+4a^3b^2}{-4a^2b} =$

x) $(a^4b^8-a^5b^7+a^6b^6-a^7b^5+a^8b^4) \div (-a^4b^4) =$

y) $(-9a^4b^2+18a^4b^3-27a^6b^4+9a^3b^2-45a^3b^6) \div (-9a^3b^2) =$

Further exercises

Further exercise 1

1. Remove the bracket!

a) $a\times(b+c)=$ b) $b\times(a+3)=$ c) $5\times(a+b)=$

d) $8\times(5+a)=$ e) $3a\times(b+7)=$ f) $7a\times(a+2)=$

g) $5a^2\times(a+6)=$ h) $7c^2\times(b+c)=$

2. Find a common factor!

a) $ab+bc=$ b) $ab+2a=$ c) $7a+7c=$

d) $27+9a=$ e) $8a+2ab=$ f) $9a^2+9a=$

g) $6a^3+24a^2=$ h) $9c^3+9c^2d=$

***3. A mixture of removing bracket and finding common factor!**

a) $5ac\times(3a+c)=$ b) $4bc^2+36b^2c=$ c) $\left(b^2+2c+b\right)\times 3b^2=$

d) $16a^2b^2+48ab^3-8ab^2=$

Further exercise 2

1. Remove the bracket!

a) $13\times(2a-3b+2)=$ b) $7\times(a-3b-2)=$

c) $a\times(-2a+b-3)=$ d) $b\times(1-2b+4a)=$

2. Find a positive **common factor!**

a) $22a^2+33-11a=$ b) $-12a+24b+36=$

c) $ab-2a-4a^2=$ d) $-3a+4ab^2+a^2=$

3. Remove the bracket!

a) $(-11)\times\left(2a-6a^2+b\right)=$ b) $(-4)\times\left(-6a+3a^2+1\right)=$

c) $(-a)\times\left(1-2a+3a^2b\right)=$ d) $(-b)\times\left(-5+2a^2-3ab\right)=$

4. Find a negative **common factor!**

a) $-21+42a-63ab=$ b) $7a^2-14ab-7b^2=$

c) $a-a^2+6ab=$ d) $-2b+3b^2-4ab^2=$

Further exercise 3

1. Calculating using (higher) powers of a!

a) $a^3\times a^4=$ b) $a^6\times a^2=$ c) $a^3\times a^7=$ d) $a\times a^4=$

2. Remove the bracket!

a) $9a\times\left(2a^2+3a-4\right)=$ b) $2x^2\times\left(x^2+3x-4\right)=$

c) $7ab\times(1-2a+b)=$

3. Find a positive **common factor!**

a) $6a-8a^2-14a^3=$ b) $2a^5+a^3-a^2=$

c) $15m^2n-20mn+10mn^2=$

4. Remove the bracket!

a) $(-2a)\times(4+3a)=$ b) $\left(-7x^2\right)\times\left(1-4x^2\right)=$

c) $(-5xy)\times\left(2x+3y^2\right)=$ *d) $\left(1+2a^3b-ab^2\right)\times\left(-6a^2b\right)=$

5. Find a negative **common factor!**

a) $-24a-20a^2=$ b) $-6x^4+2x^2=$ c) $-44a^3b+55ab^2=$

**d) $-5a^4b^4+5a^3b^5-5a^2b^6=$

Further exercise 4

1. Remove the brackets!

a) $(a+3)(a+15)=$ b) $(a+2)(a+19)=$ c) $(a+1)(a+11)=$

d) $(a+7)(a+6)=$ e) $(a+3)(a+12)=$ *f) $(a+3b)(a+7b)=$

2. Change the trinomials into the product of two brackets (Factorize!)**!**

a) $a^2+14a+48=$ b) $a^2+16a+39=$ c) $a^2+16a+48=$

d) $a^2+11a+10=$ e) $a^2+13a+36=$

**f) $a^6+11a^3b+18b^2=$ The answer to f) needs to be checked!

Further exercise 5

1. Remove the brackets!

a) $(a+2)(a+11)=$ b) $(a-2)(a+10)=$

c) $(a-5)(a+1)=$ d) $(a+4)(a+13)=$

e) $(a-3)(a+8)=$ f) $(a-7)(a+6)=$

*g) $(a+4b)(a+8b)=$ **h) $(3a-2b)(3a+11b)=$

2. Change the trinomials into the product of two brackets (Factorize!)**!**

a) $a^2+10a+21=$ b) $a^2-7a-18=$ c) $a^2+9a-36=$

d) $a^2+13a+12=$ e) $a^2+8a-9=$ f) $a^2-7a-8=$

*g) $a^2+19ab+34b^2=$ **h) $16a^2-4ab-2b^2=$

The answer to h) needs to be checked!

Further exercise 6

1. Removing the brackets and factorizing!

a) $(a-4)(a-7)=$ b) $(a-1)(a-9)=$

c) $a^2-15a+50=$ d) $a^2-3a+2=$

2. Removing the brackets and factorizing! (all cases mixed together)!Watch out for the signs!

a) $(a-4)(a+9)=$ b) $(a-2)(a-11)=$ c) $a^2+2a-35=$

d) $a^2+11a+18=$ e) $(a+1)(a+13)=$

f) $(a-13)(a+3)=$ g) $a^2-4a-45=$ h) $a^2-13a+36=$

i) $(a-3)(a+13)=$ j) $a^2+7a-8=$

*k) $(a+4b)(a-7b)=$ **l) $4a^2-2ab-30b^2=$

The answer to l) needs to be checked!

Further exercise 7

1. Examples using the three binomial formulae!

a) $(a+7)^2=$ b) $(a-12)^2=$ c) $(a+6)(a-6)=$

d) $a^2-8a+16=$ e) $a^2+2a+1=$ f) $a^2-256=$

*g) $(3a+4)^2=$ *h) $121a^2-44a+4=$ *i) $81a^2-225=$

2. Complete the binomial formulae! Watch out for the signs!

a) $(....\quad ....)^2=a^2\ +\ ....\ +\ 64$

b) $(....\quad ....)^2=a^2\ -\ 20a\quad ....$

c) $(....\ +\ ....)(a\quad ....)=....\ -\ 9$

**d) $(....\ +\ 3b)(....\quad ....)=40000a^2\ -\ ....$

Supplementary exercise: **Further exercise 8**

1. Removing the brackets and factorizing using the three binomial formulae!

a) $(b+a)^2=$ b) $(b+7)^2=$ *c) $(3b+12)^2=$

d) $(a-b)^2=$ e) $(4-b)^2=$ *f) $(5a-7)^2=$

g) $(a-c)(a+c)=$ h) $(5-c)(5+c)=$ *i) $(14a+3)(14a-3)=$

j) $x^2+2xy+y^2=$ k) $a^2+26a+169=$ *l) $196+140a+25a^2=$

m) $x^2-2xz+z^2=$ n) $a^2-28a+196=$ *o) $49-84a+36a^2=$

p) $y^2-z^2=$ q) $y^2-289=$ *r) $169x^2-289=$

Further exercise 9

1. Change into the product of two brackets!

Trinomials and binomials mixed!

a) $x^2+10x+21=$ b) $x^2+14x+49=$ c) $x^2+50x+49=$

d) $x^2-8x-33=$ e) $x^2-121=$ f) $x^2-10x-11=$

g) $x^2-20x+100=$ h) $x^2-52x+100=$ i) $x^2-100=$

j) $x^2+30x+225=$ k) $x^2+30x+200=$

l) $x^2+30x+125=$ m) $x^2-144=$ n) $x^2-70x-144=$

o) $x^2-24x+144=$ *p) $4x^2+4xy+y^2=$

*q) $9x^2+3x-3x-1=$ *r) $x^4+11x^2y+28y^2=$